

Envelopment in Resource Dependent Branching Processes and Directives of Control.

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Abstract

Resource dependent branching processes (RDBPs) were created as an attempt to model the development of human societies governed by different policies ([1], [3]). The idea was to study how societies will develop if they follow two major hypotheses, namely that people want to live and to be protected by the society in which they live, and that they will usually prefer a higher standard of living to a lower one. The arguably strongest result for RDBPs under the above hypotheses is the theorem of envelopment ([3]), saying that, in the long run, human populations, whatever their society form may be, are bound to live in an envelope spanned by two specific societies. These are the so-called weakest-first society and strongest-first society. For fixed parameters of natality, production and consumption, the first one establishes a uniform upper bound, whereas the second one presents the best possible asymptotic lower bound.

The objective of this talk is to show that the envelopment theorem, which presents, first of all, a fact of macro-economic interest, can sometimes be directly applied as a tool of control. For example, a nation with a low natality rate, may risk considerable acceleration effects towards extinction through new legislation, a fact which seems, a priori, unrelated. (This is exemplified in [2] for Germany with its new retirement legislation.) Such collateral effects may seem surprising. Nevertheless, they must be understood and controlled for. We will explain what exactly is behind such phenomena. Then we will show more generally how to exploit the survival inequality for RDBPs (intrinsic in the envelopment theorem) to obtain concrete directives for policies. It turns out that if certain parameters of a population are in specific ranges, the theorem of envelopment leads to definite directives of control in view of changing other parameters through the change of policies. We also shortly discuss new attempts of Bruss and Duerinckx to obtain "local sub-envelopes" of greater applicability.

References

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