Phase Transitions for Branching Random Walks with Various Space Dynamics

Elena Yarovaya, yarovaya@mech.math.msu.su

Department of Probability Theory, Moscow State University, Russia

Keywords: Branching processes, Random Walks, Non-homogeneous Environments, Heavy Tails, Phase Transitions, Large Deviations, Green Functions

AMS: 60J80, 60J35, 60G60, 37L60

Abstract

Models of symmetric and nonsymmetric branching random walks (BRWs) on $\mathbf{Z}^d$ with one source of branching have been studied in a variety of publications, see, e.g., [1]. In these models, the underlying random walk has finite variance of jumps. As a rule, when studying BRWs only the time-asymptotic results under fixed spatial coordinates are used. Meanwhile, for BRWs with large deviations for random walks it is urgent to know asymptotics of the transition probabilities in the situation when the spatial and temporal variables grow jointly. In [2], a new approach based on the resolvent analysis of evolutionary operators has been proposed to study a continuous model of homopolymers on $\mathbf{R}^d$ with path large deviations for Brownian motion. However, [2] does not cover the case of BRWs on $\mathbf{Z}^d$. The goal of the study is to analyze phase transitions for various continuous-time BRWs on $\mathbf{Z}^d$ with the birth and death of particles occurred at a finite set of lattice points. Symmetric BRWs with the random walk transition rates possessed heavy tails are considered among them. Such BRWs have infinite variance of jumps and, as a result, the random walk may be transient even on $\mathbf{Z}$ and $\mathbf{Z}^2$. Conditions of transience and limit theorems for the total and local numbers of particles are obtained. In the case of BRWs with large deviations an explicit analytic formula is obtained for a joint spatio-temporal asymptotics of transition probabilities, specified by the lattice Laplacian. The limit theorems on asymptotic behavior of the Green function for the transition probabilities are established.

Acknowledgements: The author is grateful to Prof. S.A. Molchanov for the seminal ideas. The research was supported by RFBR grant N 10-01-00266.

References

[1] Yarovaya, E. (2011). *Supercritical Branching Random Walks with a Single Source.* Communications in Statistics — Theory and Methods, 40, N 16, 2926-2945.

[2] Cranston, M., Koralov, L., Molchanov, S., Vainberg, B. (2009). *Continuous model for homopolymers.* J. Funct. Anal., 256, N 8, 2656-2696.