Limit models for a general class of branching processes with memory and population dependence in large populations

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Keywords: Branching processes; Age-dependence; Population-dependence; Memory-dependence; Extinction time; Tree size

AMS: 60J80

Abstract

We present a general class of multitype branching processes in discrete time with memory and population dependent individual transitions. We study the limit model of this process, as the initial population size tends to ∞, either when normalized by the current size of the whole population, or without any normalization, considering then the only rare types of the process. The first setting may be suitable, for example, for studying an epidemic propagation of a nonfatal disease, while the second setting is suitable for fatal diseases. In the first case, we show that the normalized process has the same asymptotic behaviour on the trajectories that do not extinct, as the corresponding deterministic dynamical system on individual probabilities. In the second case, we show that the limit process concerning the rare types, as the initial population size tends to ∞, may be reduced, under a saturation assumption, to a memory- dependent process with Poissonian transitions, and we study its asymptotic time-behaviour. Moreover, in the subcritical case, we give the distribution of its extinction time and of the size of its tree until extinction. In both setting (nonrare types or rare types), we give a upper bound of the error between the original process (or its transitions) and the limit. This work is an extension and generalization of [1].

References

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