Sudden death versus slow extinction of branching processes in random environment

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Abstract

A branching process $Z(n), n = 0, 1, ...,$ is considered which evolves in a random environment generated by a sequence of iid generating functions $f_0(s), f_1(s), ...$ Let $T = \min\{k : Z(k-1) > 0, Z(k) = 0\}$ be the extinction moment of the process, $S_0 = 0, S_k = \log f_0'(1) + \cdots + \log f_{k-1}'(1)$, $k \geq 1$, be the associated random walk and $\tau(m,n)$ be the left-most point of minimum of $\{S_k, k \geq 0\}$ on the interval $[m,n]$.

Assuming that the distribution of $\log f_0'(1)$ belongs to the domain of attraction of a stable law with parameter $\alpha \in (0,2]$ and zero mean if $\alpha > 1$, we show that (under the quenched approach and given $T = n \to \infty$) the process approaches the extinction moment by passing through a number of bottlenecks, when the size of the population becomes very small, and periods of recovering, when the size of the population is of order $e^{S_m - S_{\tau(0,m)}}$ if $m < \tau(0,n)$ and of order $e^{S_m - S_{\tau(m,n)}}$ if $m \geq \tau(0,n)$. In particular, the size Z_{n-1} of the population is "not too big" and, therefore, the process dies out in a "natural" way. Similar phenomenon takes place under the annealed approach if $Var \log f_0'(1) < \infty$. If, however, $Var \log f_0'(1) = \infty$ then (under the annealed approach) $\log Z_{n-1}$ is very big, namely, of order $n^{1/\alpha}$. In this case the extinction of the process at moment n may be treated as it's sudden death under very unfavorable environment.

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