# Fixation Time Estimates in Bounded Populations

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### Abstract

Markov models are used widely in population dynamics and evolution. One of such sort models are fixed size haploid models without mutations. The discreet time model consisting of $N$ particles each of which can belong to one of $N$ types. If, at some moment, the number of particles of each type is defined by $\mathbf{k} = (k_1, \ldots, k_N)$, where $k_j \in \{0, \ldots, N\}$ and $|\mathbf{k}| = k_1 + \ldots + k_N = N$, then, at the next moment, the number of particles of each type is described by the random vector (rv) $\left( \sum_{j\in K_1} \xi_j^{(N)}, \ldots, \sum_{j\in K_N} \xi_j^{(N)} \right)$, where $\xi_j^{(N)}$ is the number of offspring of the $j$-th particle, the sets $K_1, \ldots, K_N$ consist of the numbers of the particles with types $1, \ldots, N$ ($K_1, \ldots, K_N$ have $k_1, \ldots, k_N$ elements and are pairwise disjoint) and $\xi_1^{(N)} + \ldots + \xi_N^{(N)} = N$. The rv $(\xi_1^{(N)}, \ldots, \xi_N^{(N)})$ have exchangeable distributions (i.e., the birth law does not depend on their type) are i.i.d. at each step. Let $\tau_{\mathbf{k}}$ be the fixation time of the population (i.e., supplanting all particles by particles of one type) with initial configuration $\mathbf{k}$. The main result is $\mathbf{E}\tau_{\mathbf{k}} \le c\Big( (N-1)N \ln N - \sum_{j=1}^{N} (N-k_j) \ln(N-k_j) \Big)$, for some constant $c$ depends of $N$ and moments of $\xi_1^{(N)}$ [1].

Let $Z_n(m)$ bounded by level $N$ supercritical Galton-Watson process, defined as $Z_0 = m$, $Z_n(m) = \min\Big(N, \sum_{j=1}^{Z_{n-1}(m)} \xi_{j,n}\Big)$. The process $Z_n(m)$ is finite Markov chain with one absorbing state. But the extinction time for one very large and very long time we have $Z_n(m) \approx N$, as $m \approx N$ and $N \gg 1$ [1]. Empirical results shou that $\mathbf{E}\tau_{\mathbf{k}}$ from previous model equivalent to analogy parameters for the branching process.

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### References

[1] Klokov, S. A. & Topchii, V. A. (2006). *Mean fixation time estimates in constant size populations.* Siberian Mathematical Journal, V. 47, N. 6, 1042-1053.

[2] Topchii, V. A. (2009). *Extinction time for Galton-Watson processes cutting at high level.* Theory of Probability & Its Applications, V. 53, N. 1, (in print).