Limit theorems for conditioned multitype branching processes and Feller diffusions. Application to the BSE epidemic.

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Abstract

We consider a multitype continuous-time branching process, and study in the critical and subcritical cases the conditioned distribution $\mathbb{P}(X_t \in . | X_{t+\theta} > 0)$, for some $\theta \geqslant 0$. We investigate both limits as θ and t tend to ∞, as well as the commutativity between them. The first limit stands for a conditioning on non-extinction in the remote future, while the second limit is a generalization of the Yaglom distribution (obtained for $\theta = 0$). We shall investigate the same limits for a multitype Feller diffusion process, as well as the connexion between the discrete and continuous-state cases.

In terms of risk analysis, the study of the conditioned distribution $\mathbb{P}(X_t \in . | X_{t+\theta} > 0)$ for θ very large is of some interest. If the process represents the size of an infected population, this distribution corresponds indeed to the worst scenarios of the epidemic. We shall apply some of the above-mentioned results to the modelling of the propagation of BSE elaborated by C. Jacob. In [2], the authors model the incidence of clinical cases via a monotype Markovian process of order d, which can be considered as a multitype Bienaymé-Galton-Watson process having d types corresponding to the memory of the process. We would like to estimate parameters of this model in the previous setting, which would provide information about the worst scenarios of the epidemic.

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References

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