Statistical inference of critical multitype branching processes with immigration

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Abstract

We will investigate a d-type branching process $X_k = (X_k^1, \dots, X_k^d)^\top$, $k = 0, 1, \dots$, with immigration. By $\xi_{k,\ell}^{i,j}$ we denote the number of type j offspring produced by the ℓ^{th} individual who is of type i belonging to the $(k-1)^{\text{th}}$ generation. The number of type i immigrants in the k^{th} generation will be denoted by ε_k^i. Then

$$X_k = \sum_{i=1}^{d} \sum_{\ell=1}^{X_{k-1}^i} \xi_{k,\ell}^i + \varepsilon_k,$$

where $\varepsilon_k := (\varepsilon_k^1, \dots, \varepsilon_k^d)^\top$ and $\xi_{k,\ell}^i := \left(\xi_{k,\ell}^{i,1}, \dots, \xi_{k,\ell}^{i,d}\right)^\top$. Assume that $m_\xi := (E(\xi_{1,1}^{i,j}))_{1 \le i,j \le d}$ and $m_\varepsilon := E(\varepsilon_1)$ are finite. Then $E(X_k \mid X_1, \dots, X_{k-1}) = m_\xi^\top X_{k-1} + m_\varepsilon$. If m_ε is known then the conditional least squares estimator $\widehat{m_\xi}^n$ of m_ξ based on the observations $X_1, \dots, X_n$ can be obtained by minimizing the sum of squares $\sum_{k=1}^n \|X_k - m_\xi^\top X_{k-1} - m_\varepsilon\|^2$ with respect to m_ξ, and we obtain

$$\widehat{m_\xi}^n = \left(\sum_{k=1}^{n} X_{k-1} X_{k-1}^\top \right)^{-1} \left(\sum_{k=1}^{n} X_{k-1} (X_k - m_\varepsilon)^\top \right).$$

We are interested in the asymptotic behaviour of the sequence $\widehat{m_\xi}^n$, $n = 1, 2, \dots$, in the critical case $\varrho(m_\xi) = 1$, where $\varrho(m_\xi)$ denotes the spectral radius of m_ξ.

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References

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