

Elementary Proofs for Certain Extinction in Bisexual Galton-Watson Branching Processes

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Abstract

Interest in the standard Galton-Watson branching process can be motivated by a consideration of two questions. Given: a singular class of asexually reproducing individuals, what condition will guarantee certain extinction (i.e. an extinction probability of one)? (Answer: when the mean of the offspring probability distribution is less than or equal to one.) Also, if the extinction probability is known to be less than one, how can that probability be calculated? (Answer: solve the equation, the generating function of the offspring probability distribution (in terms of x) equal to x . The least positive solution will be the probability of extinction if there is one individual in the initial generation.) The mathematical arguments associated with the two answers are relatively simple and can be understood by novice probability students. These relevant questions should be asked in the context of the bisexual Galton-Watson branching process. However, the simplicity associated with asexual reproduction is lost when the model requires bisexual reproduction. A consideration of the first question with the assumption of a two-sex population will be considered in this paper. Proofs relating to this question has found in the literature are not simple and require a high level of mathematical sophistication. Perhaps, a greater interest in the bisexual model could be generated, if the proof associated with the first question, could be presented for a target audience with minimal mathematical backgrounds. That project will be attempted in this paper.