Stochastic differential equation models of population and individual growth in a random environment

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Abstract

Consider a population growing in an randomly varying environment and let $N = N(t)$ be its size at time t. We assume that the *per capita* growth rate has mean behavior $g(N)$ (a general smooth function satisfying suitable assumptions dictated by biological considerations) and is affected by a (white) environmental noise with constant noise intensity σ. We thus obtain the general stochastic differential equation (SDE) model $\frac{1}{N}\frac{dN}{dt} = g(N) + \sigma\varepsilon(t)$ or, in traditional format, $dN = g(N)Ndt + \sigma NdW(t)$, where $\varepsilon(t)$ is a standard white noise and $W(t)$ is the standard Wiener process.
For the special case of Malthusian growth ($g(N)$ constant), comparing this model with the corresponding classical branching process model, that studies demographic noise (sampling variation in births and deaths), one notices that changes in population size have a standard deviation proportional to N in the SDE model (environmental noise) and proportional to $\sqrt{N}$ in the branching process model (demographic noise).
We will examine the behavior of our general SDE model in what concerns the existence of a stationary density for the population size and in what concerns extinction. We then generalize the results to the case of density dependent noise intensities $\sigma(N)$.
Finally, we use similar models (work with Patrícia A. Filipe) to describe individual growth of animals from birth to maturity and illustrate with data on cattle weight.

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