# Random self-similar measures: "Multifractal spectra"

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## Abstract

Given a compact set $K$ ($\subset \mathbb{R}^d$), an associated scaling law produces a random number of randomly scaled copies of $K$ inside $K$. The scaling law is then used to produce even smaller copies of $K$ inside the first generation ones, and so on. The division and scaling used on each set are independent, so this is a branching process. The union of the $n$th generation sets converges as $n$ goes to infinity to a random fractal set. As well as its physical scale (its diameter, say) already described, each copy of $K$ has second scale factor, generated in the same way as its physical scale. In the limit these translate to a measure on the random fractal. The objective is to provide a description of the variation of what is called the local dimension of this measure. (The local dimension is $\alpha$ at a point when the log of the measure of a ball of radius $r$ around that point divided by the log of $r$ converges to $\alpha$.) Good results of this kind are already available when the measure is defined on the boundary (the infinite lines of descent) of the Galton-Watson tree. The main contribution in this work is dealing with the extra geometry resulting from building realisations in $\mathbb{R}^d$, extending the results in Arbeiter and Patzschke (1996).

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## References

[1] M. Arbeiter and N. Patzschke. (1996). *Random self-similar multifractals.* Math. Nachr., 181:5–42.