Applications of branching processes to a network epidemic model with casual contacts

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Abstract

Branching processes play a key role in the analysis of epidemic models. In this talk, I illustrate this by outlining some recent work with Peter Neal (University of Manchester) concerned with epidemics on random networks incorporating casual contacts (Ball and Neal [1]). I consider a stochastic SIR (susceptible $\rightarrow$ infective $\rightarrow$ removed) model for the spread of an epidemic on a finite network, having an arbitrary but specified degree distribution, in which individuals also make casual contacts, i.e. with people chosen uniformly from the population. The behaviour of the model as the network size n tends to infinity is investigated. In particular, its threshold behaviour is determined by exploiting an approximate branching process, with the approximation becoming exact in the limit as $n \rightarrow \infty$. The proportion of the population who are ultimately infected by an epidemic that becomes established is determined, using another approximating branching process, and a central limit theorem for the size of such an epidemic is obtained. Corresponding results for the epidemic without casual contacts, i.e. for the standard SIR network epidemic model, are discussed briefly. The theory is illustrated by numerical studies which demonstrate that the asymptotic approximations work well, even for only moderately sized networks, and that the degree distribution and the inclusion of casual contacts can each have a major impact on the outcome of an epidemic.

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References

[1] Ball, F.G. and Neal P.J. (2008). *Network epidemic models with two levels of mixing.* Math. Biosci., 212, 69-87.